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DATA PROCESSING AT THE GLOBAL POSITIONING SYSTEM MASTER CONTROL--ETC(U)  
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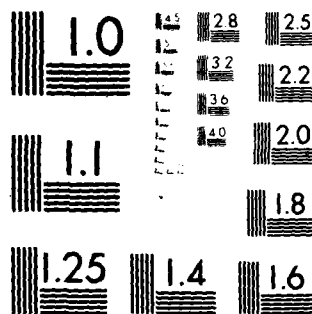
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DATA PROCESSING AT THE GLOBAL POSITIONING  
SYSTEM MASTER CONTROL STATION

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## INTRODUCTION

The mission of the GPS control segment is to collect measurements from remote unmanned monitor stations at a central location - the master control station (MCS) - to process these data in near real time, and to upload ephemeris and clock data based on this processing to each of the GPS satellites. Figure 1 is a generalized graph (timewise) of the visibility of each satellite to each of the four monitor stations. Whenever a satellite is visible to one or more monitor stations, data is being collected and transmitted via dedicated phone lines to the MCS. A navigation message can be produced at any time for any satellite or set of satellites and then uploaded at any time the satellite(s) are visible to the upload station (ULS). The ULS is colocated with the MCS. In practice, daily uploads are made nominally once a day at a time to provide optimum support for user testing at the Yuma Proving Grounds. Satellite visibility at Yuma is about the same as at Vandenberg monitor station (VMS), so a study of figure 1 will show that only about 1.75 hours of four-satellite visibility is available at Yuma for this testing each day. The start of this period is the rise of NAV 5. The desire to process some NAV 5 data prior to generating the upload conflicts with the need to provide the maximum testing time. A compromise is made, and a minimum of fifteen minutes of NAV 5 data is processed prior to generating the navigation message for this satellite. The time required to perform the calculations and prepare the navigation message for transmission is nominally 3.5 minutes, and another 2.5 minutes is required to transmit the message. Yuma ends up with about 1.25 hours of test time. Clearly, the timeline for daily preparation and transmission of navigation messages is tight. Uploads, during the past 12 month period were successfully completed according to this schedule 97.25% of the time.

Before proceeding through the total sequence of operations involved in the processing of data, several parts of the total process will be discussed separately to avoid lengthy explanations at that time.

## DATA COLLECTION SCHEDULE

The specific collection of data at each monitor station (MS) is controlled via routine messages sent to each MS from the MCS. While it is possible to manually intervene with specific instructions, generally routine messages assure that all available data is collected when one to four satellites are visible at a MS. When more than four are visible, an algorithm that resides in the MS computer is invoked to time-share the MS receiver's (X-Set) four channels with the visible satellites. The period between channel switches is selectable, and currently set at 5 minutes, which assures that with up to six satellites, data will be collected on all satellites within any 15 minute interval.

## REFERENCE EPHEMERIS

The Kalman filter employed at the MCS to process the measurement data is linearized about a reference ephemeris produced by the Naval Surface Weapons Center (NSWC) for each satellite. The reference ephemerides are produced by integrating 40 day trajectories from a set of initial conditions based on a least squares fit to smoothed GPS measurement data (nominally one week's worth) transmitted to NSWC daily by the MCS.

NSWC mails a new set of reference ephemerides <sup>to</sup> by the MCS every two weeks. The Kalman filter at the MCS produces residual state estimates to the reference values for orbital elements, solar pressure, and polar wander.

While the residual estimates produced by the MCS are not critically dependent on the accuracy of the predicted reference trajectories, providing linearity is preserved, they are the fundamental measure of the fidelity of the force models being used. Considerable effort has been expended to isolate sources of error in this respect. A major improvement was realized by modification of the solar pressure model as a result of work carried out primarily by Mr. William Feess of

Aerospace Corporation in Los Angeles. ~~Currently GPS data is being collected by additional GPS tracking stations located outside the US by DMA and it is hoped that given sufficient data, additional improvements can be made in the future.~~ Currently, reference trajectories typically run off in the along track direction (which is by far the worst case) by 5-20 meters per day. This is an order of magnitude improvement over what was originally experienced. Improved data from the monitor stations, improved processing techniques at NSWC, and improved solar pressure modeling are each responsible in part for this improvement.

### SMOOTHING

Given a range and delta range measurement from up to four satellites at each of four monitor stations, from a practical point of view, it is necessary to compact these total data for input to the Kalman filter. This is accomplished by polynomial fitting the range and delta range data for each MS/SV pair over a fifteen minute period and obtaining a single pair of measurement values (range and delta range) for each MS/SV for which there is data. The Kalman filter is cycled once each fifteen minutes providing a new set of estimates based on these smoothed data pairs. As will be clear later on, the smoothing is actually done to corrected measurement residuals. For the purpose of obtaining the fit for each 15 minute interval, the range and delta range measurement noise is assumed uncorrelated, and weights of .04 and 1 are applied for the range and delta range respectively. Figures 2 and 3 are a typical plots of range and delta range residuals respectively to the fit over a 15 minute interval. The fit is linear although provision is made for use of up to a fifth order polynomial. The fit is an iterative least squares with wild-point editing incorporated. Additionally the entire 15 minute smoothed value is rejected if the final sample standard deviation exceeds a selectable value. Nominally this is a 2 $\sigma$  edit applied globally without regard to SV or MS involved.



## KALMAN FILTER - GENERAL

The MCS Kalman filter employs the Carlson square root algorithm and estimates 6 residual orbital element epoch states, two epoch solar pressure states and three epoch clock states for each satellite. In addition it estimates two clock states for each non-master monitor station, one tropospheric scaling state for each monitor station and three polar wander states. The filter is linearized about a reference ephemeris. With the range and delta range predicted from the reference subtracted from the measurements, the result is a measurement residual ( $\delta R$ ) which is related to the residual states to be estimated ( $\delta \hat{x}_0$ ) by the linear equation:

$$\delta R_t = \left[ \frac{\partial R_t}{\partial x_0} \right]_t \delta \hat{x}_0 + v(t)$$

$$\frac{\partial R_t}{\partial x_0} = \frac{\partial x_t}{\partial x_0} \frac{\partial R_t}{\partial x_t}, \quad v(t) = \text{measurement noise}$$

The partials  $\partial x_t / \partial x_0$  that relate to the ephemeris, solar pressure, and polar wander states are provided as part of the reference ephemeris by NSWC. The non-satellite related states are estimated redundantly in each active partition. There can be up to four partitions with one to six satellites assigned to each. A satellite cannot be assigned to more than one partition. Each partition is a totally independent filter. As a consequence of a data transformation performed to decorrelate the measurement noises (range and delta range), each MS/SV measurement pair can be processed sequentially within each 15 minute Kalman cycle. Given 6 satellites and 4 monitor stations, a maximum of 24 smoothed measurement pairs would be processed each Kalman cycle if all satellites were simultaneously visible to all monitor stations.

## GPS TIME

User's of the GPS system determine the relative range to each of 4 satellites

by determining the time offset between locally generated PRN code sequences and identical code sequences generated and transmitted by the satellites. These code sequences are in fact the output of the respective clocks. The navigation message contains polynomial coefficients that predict each satellite's clock offset relative to a common time scale. This time scale is GPS time.

The MCS Kalman filter estimates three clock states for each satellite, namely, time offset, and its first and second derivatives; or equivalently phase, frequency and frequency drift. These estimates are with respect to GPS time. The filter also estimates 2 states (phase, frequency) for three of the four monitor station clocks. The fourth monitor station contains the 'master clock'. No estimates are made for this clock as it is the time base for the GPS system. Any monitor station can be designated as the 'master'. Originally the Vandenberg monitor station was designated as master, and GPS time was the same as the VMS station clock. Practical considerations have required master clock switches from time to time, and when this is done, step changes in GPS are avoided by bookkeeping the time difference between old and new master clock. GPS time is then given by the master clock adjusted by a fixed time bias maintained in the GPS data base. This bias is fixed until another master clock switch is made. No provision is made to prevent a step change in the rate of GPS time when a master clock switch is made. The magnitude of this rate change of course depends on the relative frequency offset between the cesium oscillators in the old and new master stations. Typically this is in the order of several parts in  $10^{13}$ .

#### PROCESSING SEQUENCE

Refer to figure 4 which is a flow chart of the sequence of processing steps described below. All processing as described takes place within the MCS.

Raw data arrives from each monitor station to which satellites are visible as polled by the MCS. The data consists of pseudorange and delta pseudorange measure-

ments taken each 6 seconds and time-tagged by the monitor station clocks. Differential range ( $L_1-L_2$ ) to be used for ionospheric correction and various flags indicating receiver status are also transmitted. A raw data edit based on the receiver status flags eliminates any meaningless or suspect data at the outset. Prior to this edit, a magnetic tape (RAWTRAN) is written, from which if necessary, reprocessing can be initiated in non-real time. The time tags are corrected to GPS time of signal origination at the satellite. Transit time and monitor station clock offset is determined by dividing the pseudorange measurement by the velocity of light, and an additional adjustment for satellite clock offset is made based on the current Kalman estimate of this state.

A reference range and delta range pair is computed by interpolating 15 minute data points provided in the reference ephemeris for the corrected GPS time tag of each measurement. These reference values are subtracted from the measurements to generate residual measurements. These residual measurements are the basis for estimating residual states in the Kalman filter. These residuals are then corrected for deterministic sources of measurement error as listed in figure 4.

At this point, corrected measurement residuals for each 6 second range and delta range measurement that passed the raw data edit for all active MS/SV pairs are accumulated in an ordered file for a given 15 minute interval. When all available data are collected for a given interval, the smoothing algorithm is invoked to create a single pair (range and delta range) of smoothed measurements for each MS/SV pair for that 15 minute interval. The smoothing algorithm is a weighed least squares polynomial (linear) fit to the combined range and delta range data, with the delta range weighted 25 times the range data. Individual wild-point editing is performed with iteration, and the final values are edited based on the sample variance ( $2\sigma$ ). Each pair of smoothed values, is edited again by comparison with an expected value and its uncertainty based on the last set of Kalman estimates and their covariances.

Not shown in the flow chart, is a transformation applied to the smoothed

measurement residual pairs to decorrelate the range and delta range. (In the smoothing algorithm, the smoothed value selected lies midway between the earliest and latest data point. If this is not exactly midway in the 15 minute interval, some correlation is introduced.) Decorrelation at this point permits sequential processing of the smoothed measurement residuals in the Kalman filter.

The Kalman filter updates the estimates of its residual states once each 15 minutes, and at any time, a prediction of each satellite's position and clock offset can be generated from these residual estimates together with the reference values. Refer to figure 5 for the remainder of this processing sequence.

Given Kalman residual state estimates, the satellite position can be predicted in cartesian earth-fixed coordinates by the equation shown in figure 5. To generate a navigation message say for 26 hours into the future, this equation is employed to generate 26 ninety minute sets of cartesian satellite coordinates, and a least squares fit is made for a set of 14 Keplerian-type elements to each ninety minutes. There is a half hour overlap of each set, so that a new set is to be used for each hour. The writer has employed the standard user algorithm in a hand-held TI-59 calculator to compute satellite cartesian coordinates from these elements, and obtained agreement within a few centimeters with the cartesian coordinates which formed the basis for creating the elements.

The satellite clock predictions are made by propagating the individual epoch clock state vector  $(\Delta T_0, \dot{\Delta T}_0, \ddot{\Delta T}_0)$  by the equation shown in figure 5, and calculating a set of coefficients  $(a_0, a_1, a_2)$  and a reference time  $t_0$ , for each hour (ninety minutes with overlap). The clock coefficients are each perturbed by a function which describes the effect of general relativity so that users may ignore this effect in their measurements.

## SYSTEM PERFORMANCE MEASURES

### STATE ESTIMATES

Plots are made of selected residual state estimates on a routine basis. For example figure 6 is a 30 hour plot of the Kalman estimate of current time frequency for NAV 3 - a rubidium. Figure 7 is the same for NAV 5 - a cesium. The straight line portions of the curve are during periods of non-visibility of the satellite, and represent the propagated value. Note the slope of this straight line portion for the rubidium (NAV 3) indicating a nonzero estimate for the frequency drift state. Excessive variation in estimates of clock states can be an indication of ephemeris-clock state estimate aliasing. When this occurs, reinitialization of the filter covariance matrix is used to correct the problem.

### USER RANGE ERROR

As an overall measure of system performance, a quantity called user range error (URE) is computed and plotted daily. The URE is a measure of the difference between ephemeris and clock predictions currently active in the navigation message, with those same variables according to current Kalman estimates. The assumption is that current Kalman estimates most accurately represent the truth, and the difference is a measure of the error in the navigation message. The ephemeris part of the URE is that portion of the total satellite position error projected along the line-of-sight to a selected user location (nominally Yuma Proving Grounds).

Figure 6 is a "good" URE and figure 7 is a "poor" URE. Almost all of the time, URE's range between these limits with the greater percentage looking like the former. The "total URE" is the net resulting range error due to both the ephemeris and clock errors in the navigation message. The run-off in the ephemeris component as seen in

figure 7 is for the most part the geometric effect of a more or less fixed along-track ephemeris error. Near the end of the plot, the satellite is near setting.

#### YUMA REPORTS

Daily measures of system performance are made by the test facility at the YPG. They compute relative satellite clock synchronization errors from range and delta range measurements at a known location and also measure static navigation errors. Generally these measures correlate qualitatively on a day to day basis with MCS measures.

#### ANALYTICAL DESCRIPTION OF MCS KALMAN FILTER

The basic model for the GPS Kalman Filter is

$$\begin{aligned} X(k+1) &= \Phi(k+1, k) X(k) + \omega(k+1) \\ Z(k+1) &= H(k+1) X(k+1) + \nu(k+1) \end{aligned}$$

with the usual assumptions

$$\begin{aligned} E[\omega(i) \omega^T(j)] &= Q(i) \delta_{ij} \\ E[\nu(i) \nu^T(j)] &= R(i) \delta_{ij} \\ E[\nu(i) \omega^T(j)] &= 0 \end{aligned}$$

These equations apply to estimates for vehicles within a given partition, and of course the number of states is a function of the number of vehicles represented within a partition. Each vehicle is represented in a twelve dimensional vector space, the first six states representing Keplerian orbital elements, the next three solar pressure (two used) states, and the last three clock terms. There is also a thirteen dimensional monitor-station state space to represent the four monitor-stations currently used by GPS for data collection. There are two clock states for each monitor station except a distinguished station, designated master

a-posteriori covariance from the fitting process.

The measurement update for the system is  $K (Z - \bar{H} X)$  where  $Z$  is vector of the form  $(Z_{11}, Z_{21}, \dots, Z_{s1}, Z_{12}, Z_{22}, \dots, Z_{s2}, \dots, Z_{14}, Z_{24}, \dots, Z_{s4})$  (i.e., the processing is done through vehicles for a fixed monitor station), each  $Z_{ij} = (r_{ij}, \Delta r_{ij})$ ,  $K$  being the gain computed as  $K = (\bar{H} P \bar{H}^T + R)^{-1}$ , with  $P$  the covariance updated through the time update, and  $H$  being the measurement matrix  $\bar{H} = (\bar{H}_{11}, H_{21}, \dots, \bar{H}_{s1}, H_{12}, \dots, \bar{H}_{s4})$ , each  $\bar{H}_{ij} = \begin{pmatrix} H_{ij} \\ \Delta H_{ij} \end{pmatrix}$  with  $H_{ij}$  the measurement matrix for the  $i$ th vehicle and  $j$ th monitor station. (This is the standard algorithm filtering. Actually, the GPS system uses the Carlson square root algorithm, but for our purposes here this is unessential).

Each  $\bar{H}_{ij}$  is a function of time tags based on the data smoothing during the measurement interval. If  $t_{mij}$ ,  $t_{eij}$ , and  $t_{sij}$  represent the mid-point, end-point and start point of the smoothing interval, then  $H_{ij} = H_{ij}(t_{mij})$ ,  $\Delta H_{ij} = H(t_e) - H(t_s)$ . Furthermore, each of the measurement matrices can be factored as  $\bar{H}_{ij}^s + \bar{H}_{ij}^m$  where  $\bar{H}_{ij}^s$  relates only to vehicle states and  $\bar{H}_{ij}^m$  only to the  $j$ th monitor station states.

Because of the assumption that all measurements between vehicle and monitor station are independent, the measurement update can be processed sequentially by satellite-monitor station pair. This goes as follows:

If the update were done as a full matrix process (ignoring the square-root algorithm) the update actually processed is:

$$\hat{X} = X_0 + K (Z - H X)$$

$$\text{where } K = P H^T (H P H^T + \Sigma)^{-1}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & & & \\ & \Sigma_{21} & & \\ & & \ddots & \\ & & & \Sigma_{14} & \dots & \\ & & & & \ddots & \\ & & & & & \Sigma_{s4} \end{bmatrix}$$

Thus, each measurement update only involves the matrix  $\Sigma_{ij}$ . Using this estimate of the measurement noise for the  $(ij)$  data pair, a decorrelation of range and delta-range is accomplished by transforming to variables  $\tilde{Z}_{ij}, \tilde{H}_{ij}$  using  $\tilde{Z}_{ij} = \Sigma_{ij}^{-1/2} Z_{ij}$ ,  $\tilde{H} = \Sigma_{ij}^{-1/2} \bar{H}_{ij}$ , where  $\Sigma_{ij}^{-1/2}$  is the lower-diagonal

Cholesky square root of  $\Sigma_{ij}$ . If  $\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$  then  $\Sigma^{-1/2} = \begin{bmatrix} 1/\sigma_1 & 0 \\ -\rho & \frac{1}{\sigma_1(1-\rho^2)} \sigma_2 \end{bmatrix}$

The decorrelated variables have a measurement noise covariance equal to  $2 \times 2$  identity matrix, which allows the range and delta-range for a given vehicle-monitor station pair (  $i, j$  ) to be processed sequentially. (For more information regarding the technique of decorrelating, as well as details regarding various square-root algorithms and sequential processing, see Factorization Methods for Discrete Sequential Estimation, Gerald J. Bierman, Academic Press, 1977.)

Assuming the decorrelations have been carried out, the sequential processing proceeds by cycling through the following set of equations for each vehicle-monitor station data pair:

$$\begin{aligned} K_i(j+1) &= P_{i-1} h_i / (h_i P_{i-1} h_i^T + 1) \\ X_i &= X_{i-1} + K_i(j+1) (Z_i - h_i^T X_{i-1}) \\ P_i &= (I - K_i h_i^T) P_{i-1} \end{aligned}$$

where  $P_0$  and  $X_0$  are the a-priori covariances and states after time update,  $Z_i$  represent measured range and measured delta-range (processed one after the other) and the final value update after processing all available data is the value used in the next time-update. Note that each gain  $K_i$  has a scalar in its denominator. Also note that this update gives a new estimate of the total state due to the (  $i, j$  )<sup>th</sup> range and delta-range. Processing, as noted before, cycles through vehicles per monitor station.

Another way to view the processing is by examining the measurement update process globally. Using the partitioning of the state space outlined above,  $PH^T$  can be calculated as a matrix with  $12 \times 2$  matrix coefficients, further subdivided by monitor stations, so that  $PH^T = [PH_1^T, PH_2^T, PH_3^T, PH_4^T]$ , with each  $PH_k^T$  of the form

$$\begin{bmatrix} (P_{11} \bar{H}_s^1 + P_{1m} \bar{H}_m^K) (P_{12} \bar{H}_s^2 + P_{1m} \bar{H}_m^K) \cdots (P_{15} \bar{H}_s^5 + P_{1m} \bar{H}_m^K) \\ (P_{12} \bar{H}_s^1 + P_{2m} \bar{H}_m^K) (P_{22} \bar{H}_s^2 + P_{2m} \bar{H}_m^K) \cdots (P_{25} \bar{H}_s^5 + P_{2m} \bar{H}_m^K) \\ (P_{31} \bar{H}_s^1 + P_{3m} \bar{H}_m^K) (P_{32} \bar{H}_s^2 + P_{3m} \bar{H}_m^K) \cdots (P_{35} \bar{H}_s^5 + P_{3m} \bar{H}_m^K) \\ (P_{m1} \bar{H}_s^1 + P_{mm} \bar{H}_m^K) (P_{m2} \bar{H}_s^2 + P_{mm} \bar{H}_m^K) \cdots (P_{m5} \bar{H}_s^5 + P_{mm} \bar{H}_m^K) \end{bmatrix}$$



station, for which no clock states are estimated (its clock becomes the system clock). There is also a state for a tropospheric correction at each monitor station and three polar wander states which are common to all the monitor stations.

This means that a partition represents a state-space of dimension  $12S + 13$ , where  $S$  is the number of vehicles in a partition. In the GPS system,  $1 \leq S \leq 4$ .

The estimated states are residual corrections. The ephemeris portion is linearized about a nominal reference trajectory, clock states are relative to GPS time, and all states are referenced to an epoch time (which is different for clock and ephemeris states).

If  $V$  represents the state space for a single partition, then  $V$  is decomposed into a direct sum of subspaces  $V_1 \oplus \dots \oplus V_S \oplus V_m$ , with one sub-space for each vehicle and a single subspace for the monitor station states.

The state transition matrix  $\Phi$  is blocked to accommodate the above scheme as

$$\Phi = \begin{bmatrix} \Phi_1 & & & 0 \\ & \Phi_2 & & \\ & & \ddots & \\ 0 & & & \Phi_S & \\ & & & & \Phi_m \end{bmatrix}$$

with  $\Phi V = \Phi_1 V_1 + \dots + \Phi_S V_S + \Phi_m V_m$ , each  $\Phi_i$  leaving  $V_i$  invariant. Because of this, the time update is accomplished independently for each  $V_i$ . However, since the filter is an epoch state filter, the process noise is calculated via  $\hat{P} = P + \Phi^{-1} P \Phi^{-T}$  where  $P$  is the a-priori covariance matrix for the filtering error and  $\hat{X} = X$ ,  $X$  being the a-priori state estimate.

The measurement update is processed for range and delta-range measurements between a given vehicle and a given monitor station. The two measurements are assumed independent and are collected every six seconds for a 15 minute interval. This data is smoothed through a linear weighted least-squares estimate, with the weighting matrix a fixed diagonal matrix weighting delta-range five times as heavily as range. The output of this process is a pair  $[\tau, \Delta\tau]_{ij}$  of range and delta-range residuals between the  $i^{th}$  vehicle and  $j^{th}$  MS, together with  $\sum i_j$ , the

with each of  $P_{ij} \bar{H}_S^j + P_{im} \bar{H}_M^K$  a  $12 \times 2$  matrix and  $P_{mi} \bar{H}_S^i + P_{mm} \bar{H}_M^K$  a  $13 \times 2$  matrix.

If one multiplies the first row (of  $12 \times 2$  matrices) times the measurement vector

$((\tilde{r}_{11}, \Delta\tilde{r}_{11}), (\tilde{r}_{12}, \Delta\tilde{r}_{12}), \dots, (\tilde{r}_{14}, \Delta\tilde{r}_{14}))$ , the effect will be the total update of

the first vehicles states through all the data. This is, more simply,

$$\hat{X}_i = \sum_{K=1}^u \left( \sum_j P_{ij} \bar{H}_S^j + P_{im} \bar{H}_M^K \right) \begin{pmatrix} \tilde{r}_{jK} \\ \Delta\tilde{r}_{jK} \end{pmatrix}$$

where  $(\tilde{r}_{jK}, \Delta\tilde{r}_{jK})$  is the measurement residual,  $P_{ij}$  is the submatrix of  $P$

representing the covariances between vehicle  $i$ 's states and vehicle  $j$ 's states,  $P_{im}$

is the cross-correlation matrix for the  $i^{th}$  vehicle and the monitor station  $K^{th}$

states. From this calculation one can determine the separate effects on the vehicle  $i$  due to data from vehicle  $j$  and monitor station  $K$  through the covariance matrix

$P$ . By stripping off the appropriate covariances and measurement matrix values each

of these computations can be examined individually. The term  $P_{ij} \bar{H}_S^j$  relates the  $j^{th}$

vehicle states to the  $i^{th}$  vehicle's states, while  $P_{im} \bar{H}_M^K$  relates the  $K^{th}$  monitor station states to the  $i^{th}$  vehicles states.

(The last "row" of  $13 \times 2$  matrices produces the monitor station estimates based on all the data.)

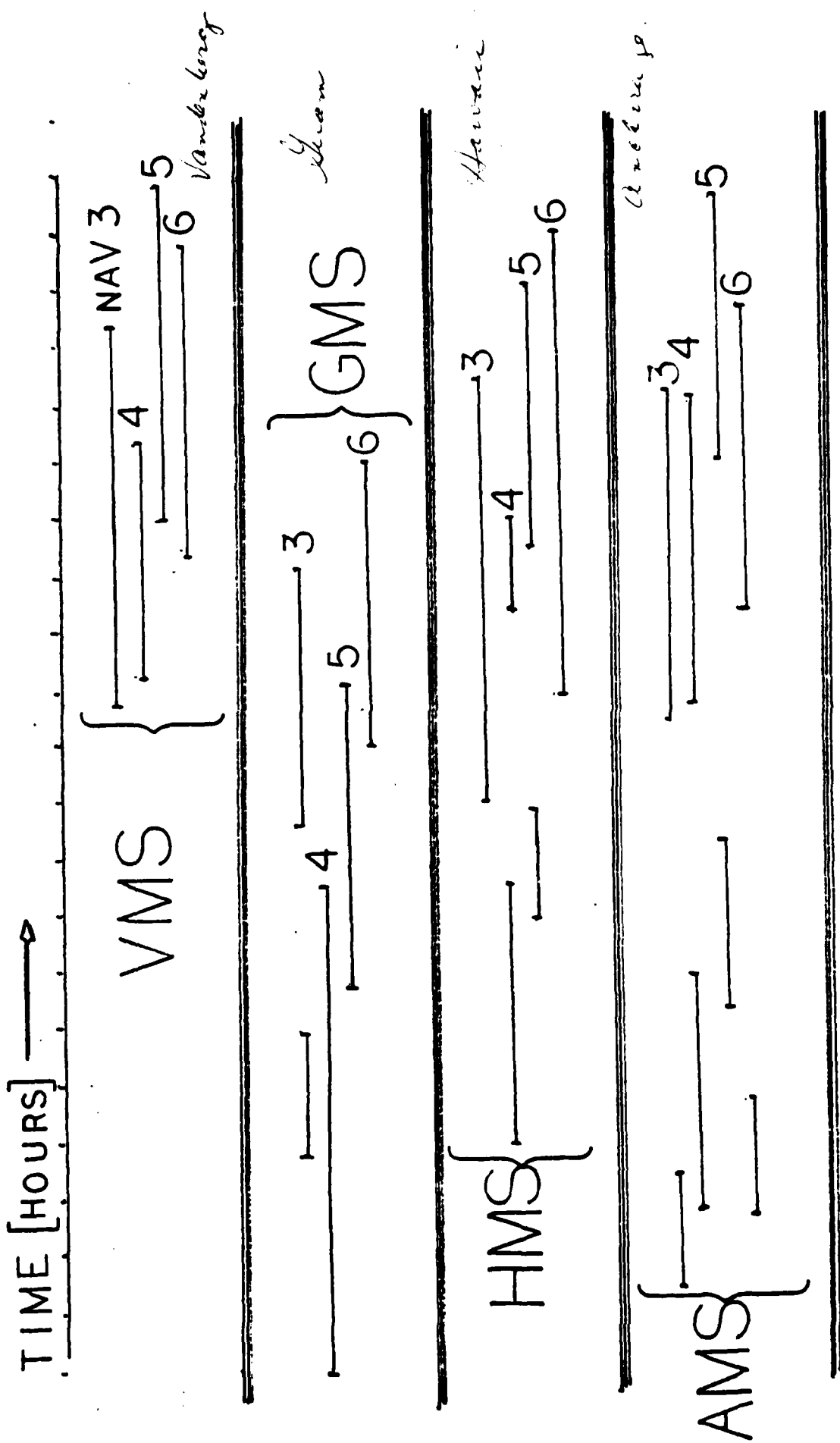
To tie this in with the sequential processing, notice that the sequential update operates on one pair  $(\tilde{r}_{ij}, \Delta\tilde{r}_{ij})$  at a time (before decorrelation, when the pairs are separated), taking a column of dimension two in width and multiplying it by the appropriate pair. This gives the effect on all the state estimates due to the appropriate pair  $(\tilde{r}_{ij}, \Delta\tilde{r}_{ij})$ . (For example, column one is multiplied by  $(\tilde{r}_{11}, \Delta\tilde{r}_{11})$ , column two by  $(\tilde{r}_{21}, \Delta\tilde{r}_{21})$  etc.) Thus, the sequential processing can be thought of as column processing. The decorrelation affected with  $\sum_{i,j}$  is to allow sequential processing of  $\tilde{r}_{ij}, \Delta\tilde{r}_{ij}$  separately.

*Varnum + Chaffee papers*

## CONCLUSIONS

The Phase I GPS has exhibited excellent performance as a real-time navigation system. It remains to be seen if accuracies equivalent to TRANSIT are possible for geodetic applications. To a large extent this may depend on the development of optimum processing strategies with multiple passes or multiple satellite data. It is not clear at this time whether precise ephemerides can be produced that are significantly better than the current navigation data transmitted from the satellites.

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# VISIBILITIES

FIGURE 1

# RMS OF PSEUDO-RANGE RESIDUALS FOR AMS AND SU NO. 5

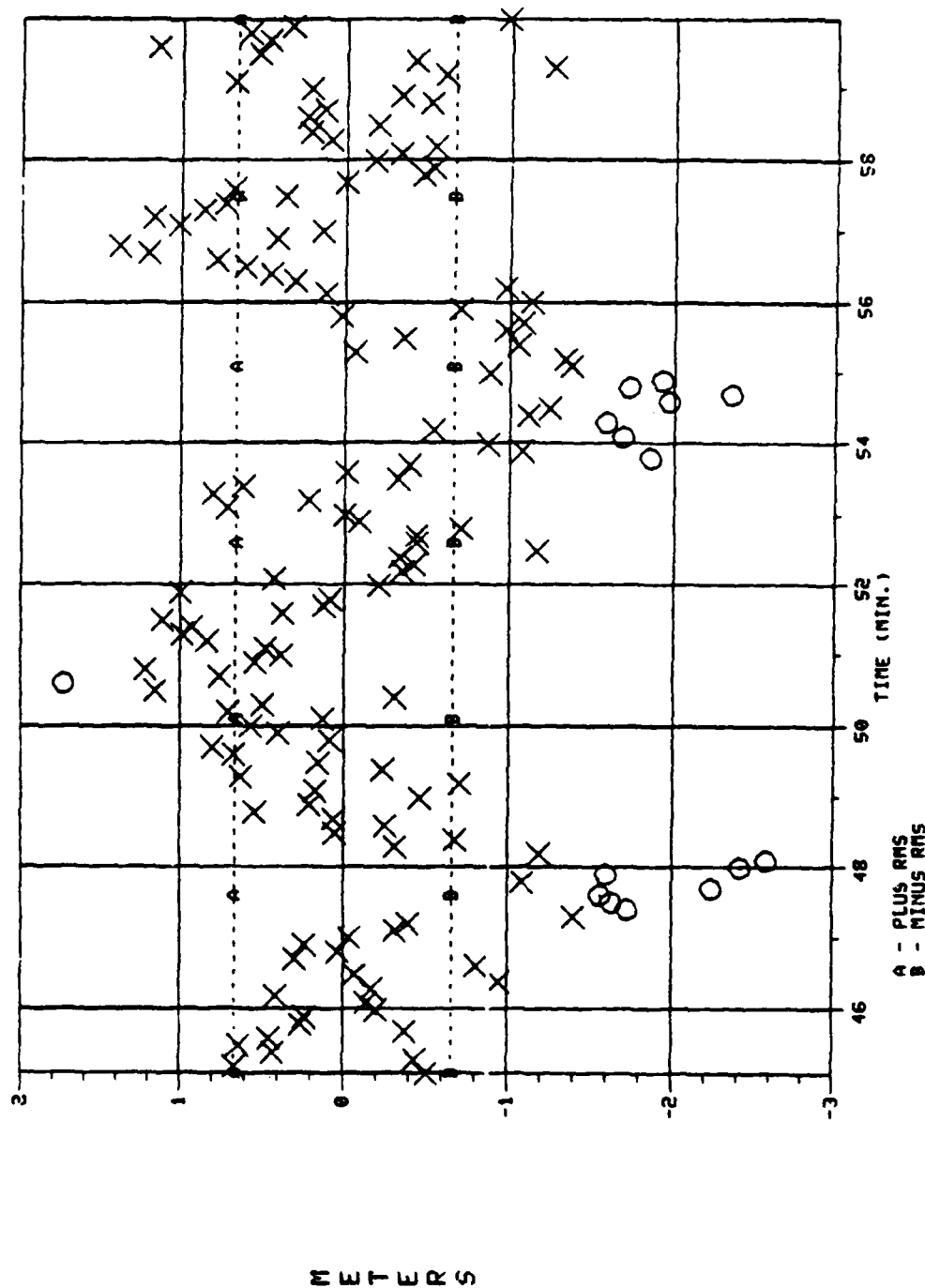


FIGURE 2

# RMS OF DELTA PSEUDO-RANGE RESIDUALS FOR AMS AND SU NO. 5

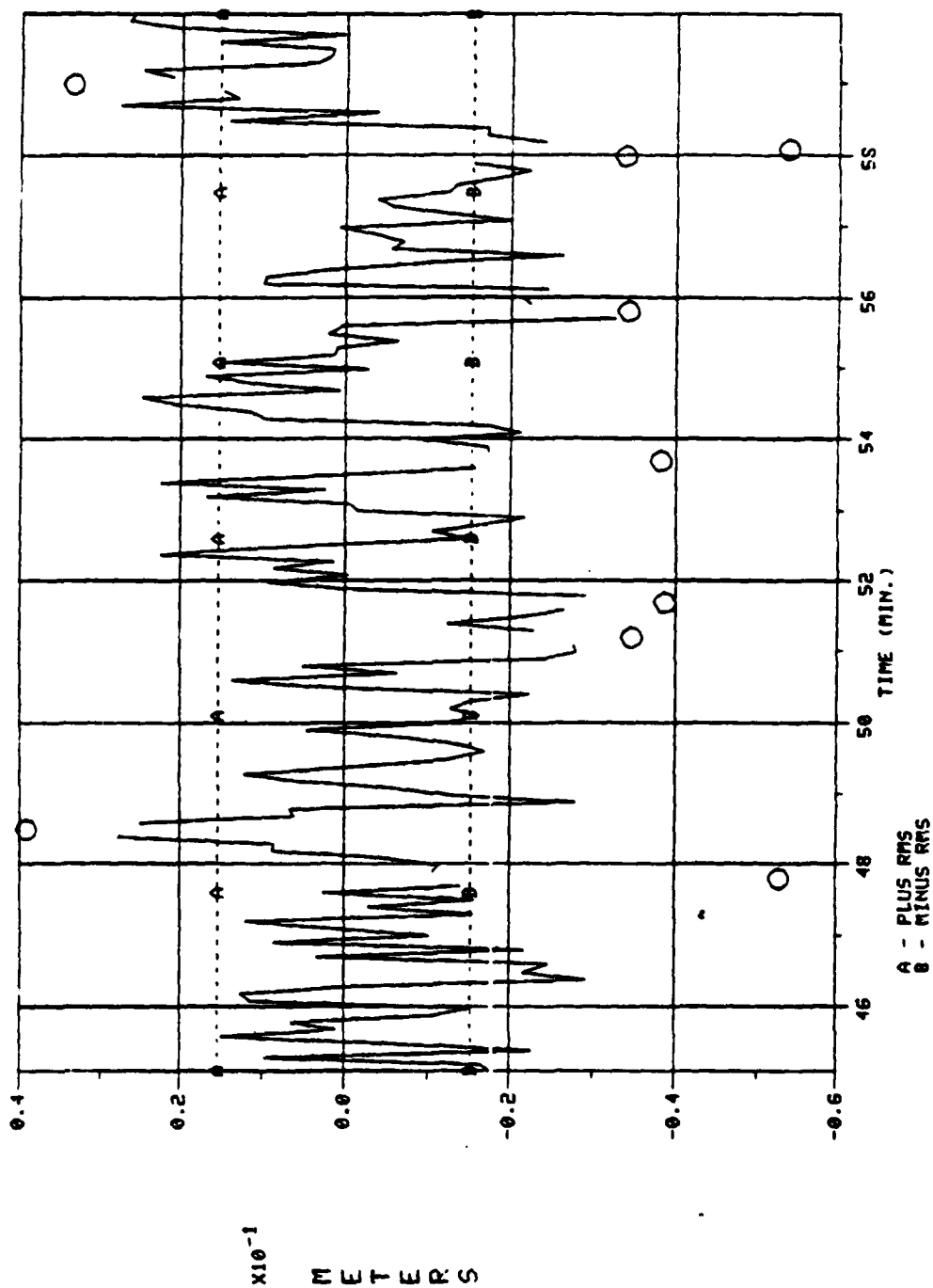


FIGURE 3

# MCS PROCESSING FLOW CHART

## INCOMING MEASUREMENTS

$R_t (ms_i, sv_i)$   
 $\Delta R_{\Delta t}$

REF  
EPHEMIS

TIME  
TAG  
CORRECTION

EDIT

$$\begin{aligned} \Delta R &= R_t - R_{t, REF} \\ \Delta \Delta R &= \Delta R_t - \Delta R_{t, REF} \end{aligned}$$

KALMAN  
FILTER

EDIT

SMOOTH

CORRECTIONS

1 PAIR ( $\Delta R, \Delta \Delta R$ ) DATA  
POINTS PER 15 MIN  
PER MS/SV

- IONOSPHERE
- TROPOSPHERE
- RELATIVITY
- EARTH ROTATION
- ANTENNA OFFSETS

RESIDUAL STATES

- SV {
- 6 ORBITAL ELEM
  - 3 SV CLOCK STATES
  - 2 SOLAR PRESSURE
- MS {
- 2 MS CLOCK STATES
  - 1 TROPO STATE
  - 3 POLAR WANDER STATES

FIGURE 4

## GENERATION OF EPHEMERIS COMPONENT AT NAV MESSAGE

### KALMAN RESIDUAL STATES ( $\delta X_0$ )<sup>I</sup>

- ORBITAL ELEMENTS
- SOLAR PRESSURE
- POLAR WANDLER

### REFERENCE EPHEMERIS

- REFERENCE POSITION ( $X_t^{EF}$ )
- PARTIAL DERIVATIVES ( $\frac{\partial X_t}{\partial X_0}$ )<sup>I</sup>

$$X_t^{EF} = X_t^{REF} + (ABCD)^I \left( \frac{\partial X_t}{\partial X_0} \right)^I (S X_0)^{EF} \Rightarrow (u, y, z) (90 \text{ MIN})$$

LEAST SQUARES FIT

14 ELEMENTS

FIGURE 5



# GENERATION OF SV CLOCK COMPONENT

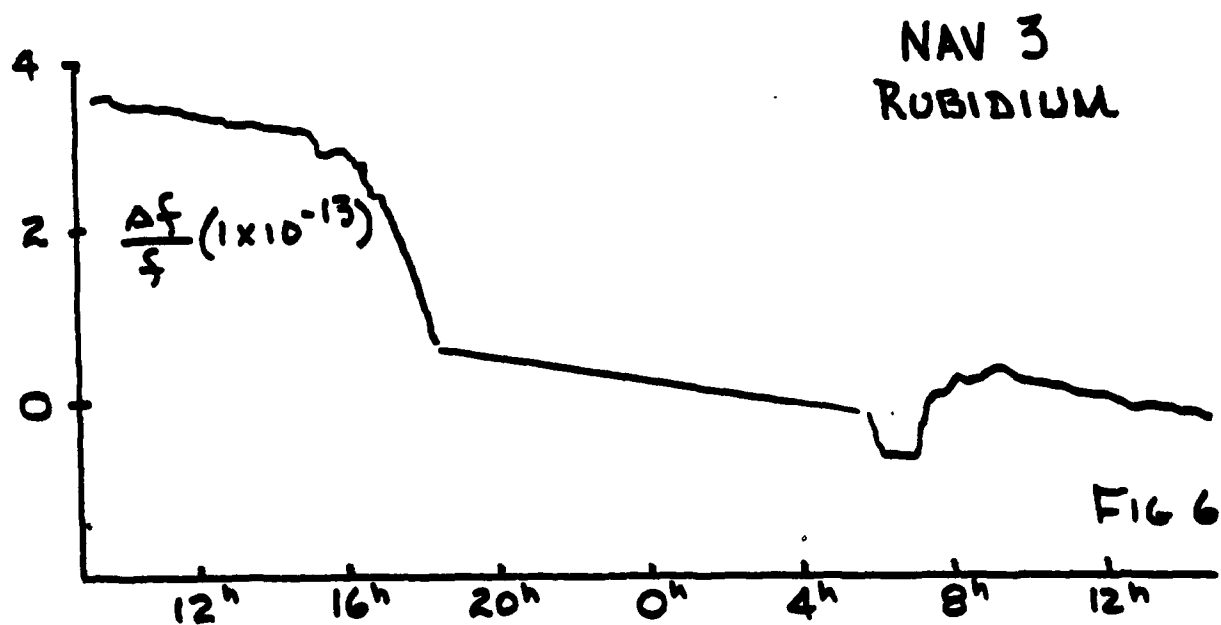
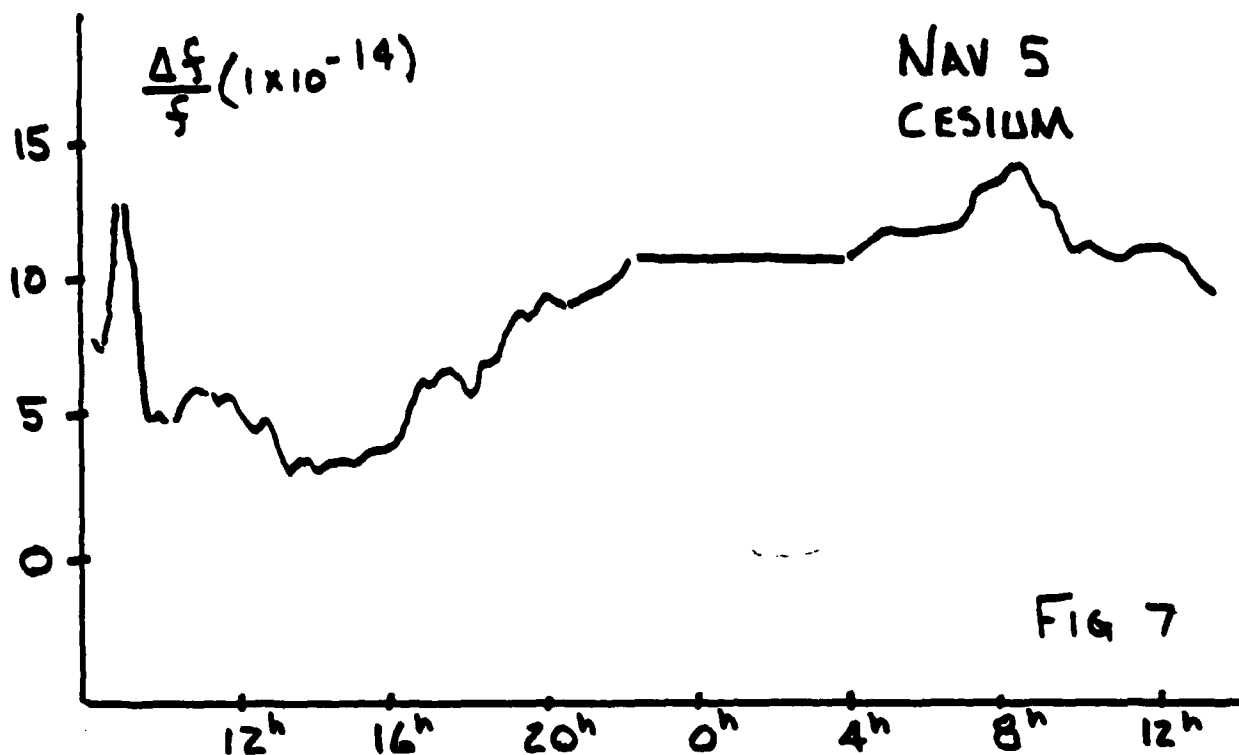
## OF NAVIGATION MESSAGE

### KALMAN SV CLOCK STATES

$$\left. \begin{array}{l} \Delta T_o \\ \dot{\Delta T}_o \\ \ddot{\Delta T}_o \end{array} \right\} \Delta \mathcal{P}_t = \Delta T_o + \dot{\Delta T}_o(t-t_o) + \frac{1}{2} \ddot{\Delta T}_o(t-t_o)^2$$

$$\rightarrow (a_o, a_1, a_2)_{90 \text{ MINUTES}}$$

FIGURE 5 (Cont)



# USER RANGE ERROR FOR SU NO. 9

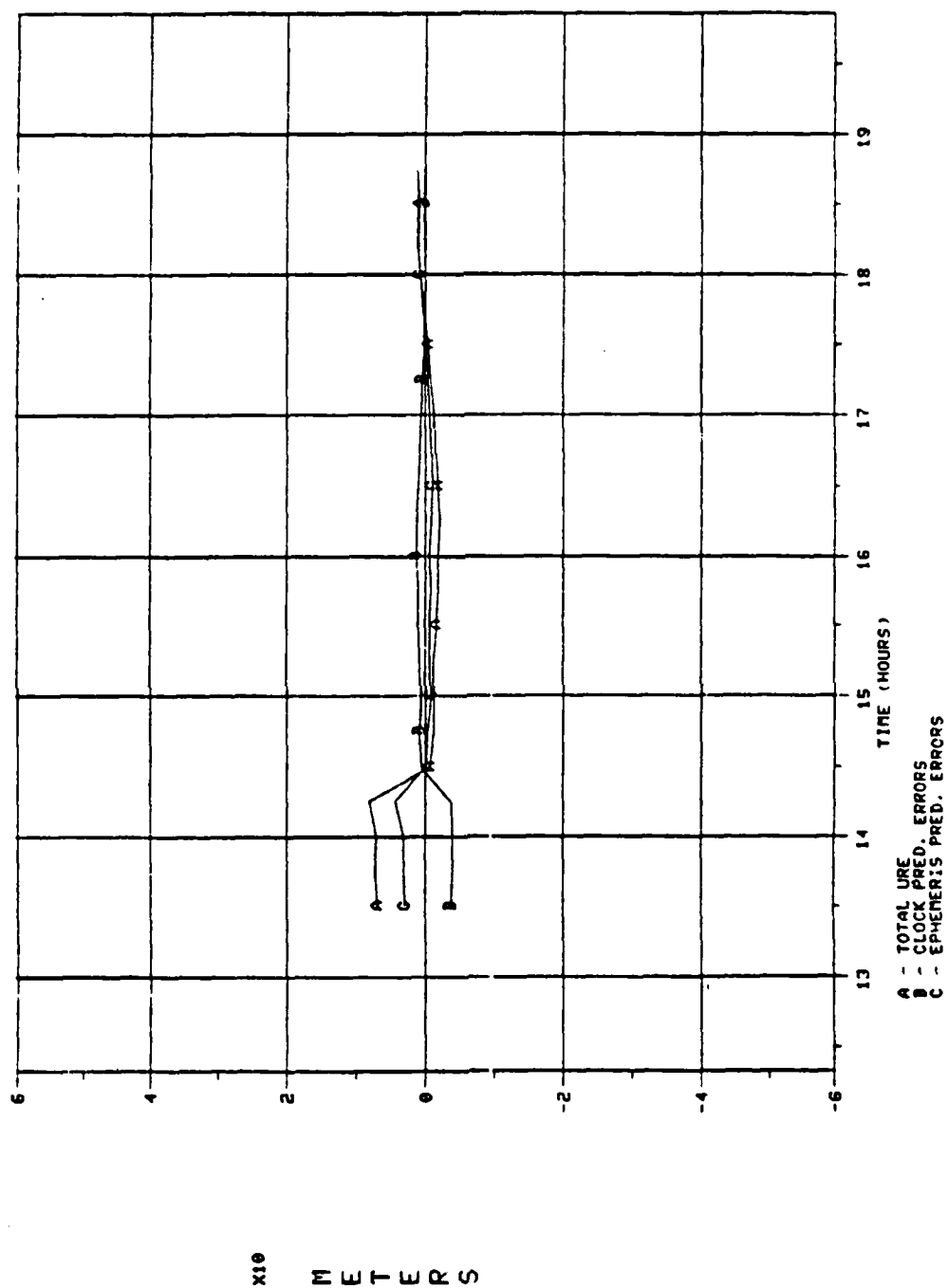


FIGURE 8

# USER RANGE ERROR FOR SU NO. 5

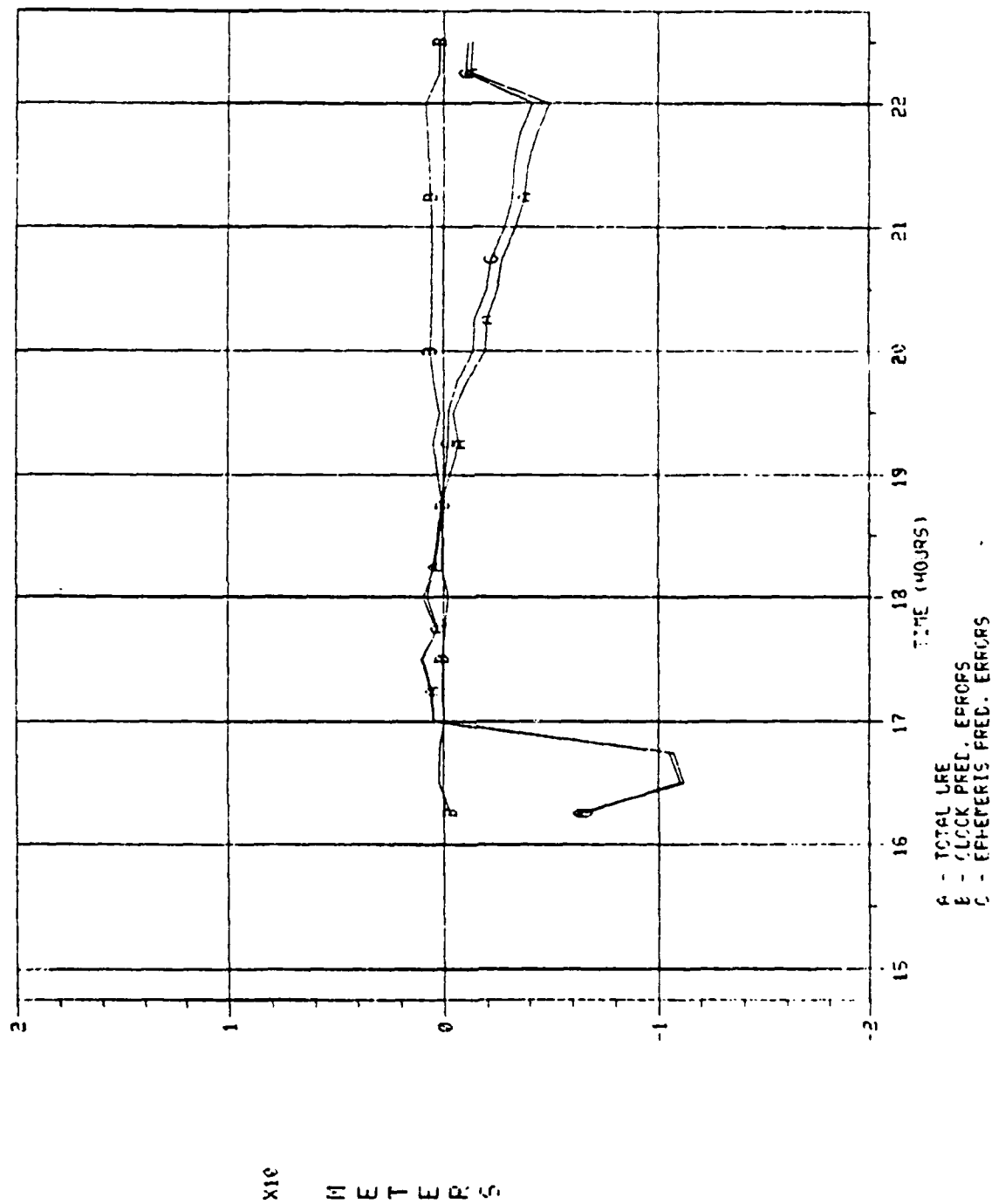


FIGURE 9

